Comparison Of Various Practical Groundwater Flow Considerations By FEM Simulation

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Abstract: Finite element groundwater modelling is extensively used for a wide range of operations for aquifer management. Present studies mentioned that for forecasting water levels, to increase natural replenishment by selective pumping, to investigate optimal well spacing for withdrawing groundwater from heterogeneous anisotropic aquifers, system simulation for confined and unconfined aquifers is practiced in many parts of the world today. Further extended application may also involve its use for artificial recharge projects, economizing the canal lining, sea water intrusion influence and to predict the spread of contaminants in polluted aquifer along with remediation strategies. However most of the works involve simplification of reality in river simulation, aquifer recharge representation, assessment of influence of other boundary conditions and aquifer heterogeneity. In the present study, a fourteen zone confined aquifer is considered. Groundwater head distribution in the flow domain is considered for constant and linearly varying river head conditions. A case of misrepresented inflow boundary as impervious boundary is discussed. Time variant groundwater heads are examined for homogeneous-isotropic, homogeneous-anisotropic, heterogeneous – isotropic aquifers and compared with more practical heterogeneous-anisotropic system properties. Study found that these are important considerations for an effective simulation of confined aquifers to realise practical conditions.

Keywords: Actual Flow Scenario, FEM Simulation, Modelling, Practical Parameter Study.

I. INTRODUCTION

A numerical groundwater flow model is a tool that simulates a real groundwater flow environment with governing equations and boundary conditions. Numerical models based on either finite difference or finite element methods (FEM) are commonly adopted for the simulation of groundwater flow. For predictive purpose, modelling performs prediction of the events that could happen in the future. For interpretive purpose, modelling is used as a framework for studying system dynamics and/or organizing field data. For generic purpose, modelling is used to analyse flow in hypothetical hydrogeologic systems. It can be very useful to help frame regulatory guidelines for a specific region, to reduce complexity and enhance convenience in modelling. In reality the flow through most porous media is three dimensional, but most aquifers are one to two orders of magnitude thinner in the vertical direction than in the horizontal direction. Therefore many groundwater flow problems can be approximated mathematically as two dimensional horizontal flow [1]. Many aquifers are analysed for synthetic flow conditions involving homogeneous isotropic system properties. Similarly river is often simulated as a constant boundary head, whereas the fact remains that no river can flow with a constant head which normally varies throughout a year. Aquifer recharge is another important parameter which is treated as constant (equal to average annual recharge) which may be far from actual recharge values. Heterogeneity and anisotropy of the aquifers is normally present in many aquifers and its actual representation makes the modelling scenario complex and difficult to resolve. Therefore temporal and linear river head variation, the rain induced surface recharge effect, the recharge variation depending on the transmissivity and storativity, which in turn influence the hydraulic head of the aquifer domain are important considerations. In this context this study aims at comparing the modified input data influence for practical groundwater flow considerations in the aquifer domain considered in [4].

II. THEORETICAL CONSIDERATION AND PROBLEM FORMULATION

The finite element method is a numerical technique for solving the problems governed by partial differential equation. The method utilizes an integral formulation to generate system of equations and uses continuous, piecewise, smooth functions for approximating the unknown quantity [6]. A system of simultaneous equations through an integral formulation is generated which when solved gives the value of the unknown field variables (ground water head) at discrete locations in the domain. FEM utilizes a continuous piecewise smooth function to approximate the unknown quantities. These two characteristics make FEM different from other numerical procedures [7]. Researches over the world have shown its potential to incorporate irregular and curved aquifer boundaries, anisotropic and heterogeneous aquifer properties and sloping soil and rock layer into the finite element numerical model. This makes it superior to finite difference method, another principle numerical technique used to model ground water flow, as the latter works best for rectangular and prismatic aquifers of uniform composition [3].

The governing equation describing the flow in a two dimensional inhomogeneous confined aquifer for time variant conditions is given as [2]:

$$\frac{\partial}{\partial x}\left[T_x\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T_y\frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} + Q_W\delta(x-x_i)(y-y_i) - q \tag{2.1}$$

Following initial conditions are used

$$h(x,y,0) = h_0(x,y) \qquad x,y \in \Omega \tag{2.2}$$

Dirichilet boundary condition is given by

$$h(x,y,t) = h_1(x,y,t) \qquad x,y \in \partial\Omega_1 \tag{2.3}$$

Neuman boundary condition considered is

$$T\frac{\partial h}{\partial n} = q(x,y,t) \qquad x,y \in \partial\Omega_2 \tag{2.4}$$

Where, *h(x,y,t)*= Piezometric head (m)

$T_x(x,y)$, $T_y(x,y)$= Transmissivity (m^2/d) along the x and y principal axes

S = Storage coefficient (dimensionless)

x,y = Horizontal space variables (m)

Q_W = Source or sink function

($-Q_W$ = Source, Q_W = Sink) ($m^3/d/m^2$)

t = Time in days

Ω = The flow region

$\partial\Omega$ = The boundary region

$(\partial\Omega_1 \cup \partial\Omega_2 = \partial\Omega)$

$\frac{\partial h}{\partial n}$ = Normal derivative

$h_0(x,y)$ = Initial head in the flow domain (m)

$h_1(x,y,t)$= Known head value of the boundary head (m)

$q(x,y,t)$ = Known inflow rate (m/d)

δ is Dirac delta function

= 1 if $x=x_i, y=y_i$

= 0 if $x \neq x_i, y \neq y_i$

The entire domain is discretized into a set of triangular elements whose shape is defined by a set of discrete points called nodes specified throughout the aquifer domain. The principle idea of FEM is to replace the exact continuous solution of the partial differential equation by a piecewise continuous solution. As triangular elements involve simpler integration and are computationally economical they are preferred over rectangular and quadrilateral elements.

First of all a trial solution (x,y,t) is defined as,

$$\hat{h}(x,y,t) = \sum_{t=1}^{NP} h_L(t) N_L(x,y) \tag{2.5}$$

Where h_L is the unknown head, N_L is the known basis function at node L and NP is the total number of nodes in the problem domain. To determine NP value of h_L a total of NP equation are required. To obtain these conditions in Galerkin's method the residuals weighted by each of the basis function are forced to be zero when integrating over the entire domain. Thus,

$$\iint_\Omega \left[\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right)\frac{\partial}{\partial y}\left[T_y\frac{\partial h}{\partial y}\right] - Q_W - S\frac{\partial h}{\partial t} + q\right] N_L(x,y)dxdy = 0 \tag{2.6}$$

After application of Galerkin's technique, the system of equation generated can be given as

$$[G]\{h_L\} + [P]\left\{\frac{\partial h_L}{\partial t}\right\} = \{F_L\} \tag{2.7}$$

The matrix [G] is the conductance matrix since it involves the terms containing aquifer transmissivity and element configurations, while the matrix [P] and [F_L] are the storage matrix and nodal recharge or discharge vector [8].

Forward finite difference scheme performs better than finite element method for the time derivative term [5]. Hence, to account for the time derivative an implicit finite difference approximation was used. Thus, for two successive time intervals t and t+Δt where Δt is the time step, Eq. (2.7) can be written as

$$[G]\{h_L^{t+\Delta t}\} + \frac{1}{\Delta t}[P]\{h_L^{t+\Delta t} - h_L^t\} = \{F_L\} \tag{2.8}$$

Rearranging the above it gives,

$$\left\{[G] + \frac{1}{\Delta t}[P]\right\}\{h_L^{t+\Delta t}\} = \frac{1}{\Delta t}[P]\{h_L^t\} + \{F_L\} \tag{2.9}$$

Here [G] and [P] need to be assembled only once for the whole problem but the system of linear equations represented by Eq. (2.9) must be solved at each time step. The solution of Eq. (2.9) will give the unknown groundwater head values at a new time step. Presently time step size of 1 day was used, however sensitivity analysis suggested little difference in simulated head values for different time step size (Δt =) 0.5, 1, 2, and 5 day respectively. This suggests the robustness of the present FEM algorithm.

Initially confined aquifer domain with thickness of 40m in the heterogeneous and isotropic condition is considered for the simulation. The area is 11.527 km^2 and is bounded by a reservoir, a river and two impervious boundaries as shown in figure 1. At the south east boundary a river flows having a constant head of 82m and towards the northern boundary there is a constant head (100m) reservoir, both hydraulically connected to the aquifer. There are impervious boundaries along the north eastern and north western boundary of the flow region. The domain also includes an irregular lake with a known head of 90m. Fourteen zones of transmissivity are considered in the domain as shown in Table 1 and a typical section along X-X is shown in figure 2. Table 1 also includes a multiplying factor (e) for an aquitard recharge used later in the study for scenario III.

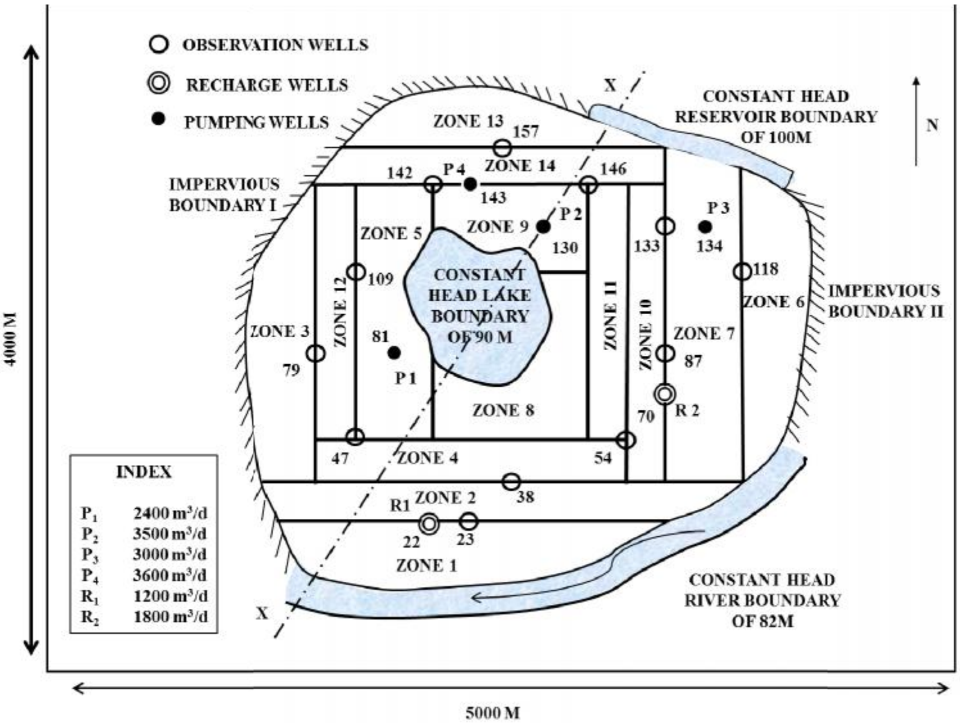


Figure 1. Plan view of Aquifer Domain

The aquitard recharge rate of 0.0001519 m/d and the transmissivity value dependant on the soil type in the domain are taken accordingly from Central Groundwater board, Maharashtra. Storage coefficient of the aquifer is 0.0004. The chosen example represents the field-applicable conditions and flow region under existing complex boundaries [4]. The impervious boundaries on the east and west may represent impermeable granite formations, intersecting faults, hillock or a water divide. Four pumping wells P_1, P_2, P_3 and P_4 having pumping rates of 2400, 3500, 3000, 3600 m^3/d at nodes 81 ,130, 134 and 143 respectively are considered. Two recharge wells R_1 and R_2 with injection rate of 1200, 1800 m^3/d at nodes 22 and 77 respectively are also present in the region under study.

The problem is simulated using the FEM model. The domain discretization using the FEM methods is shown in Figure 3. The entire aquifer domain was discretized into 296 elements made up of 174 nodes using triangular linear elements for FEM method. In this study, three scenarios representing the various practical groundwater flow considerations are analysed for evaluating its effect on the groundwater domain by FEM simulation. Two sections one in horizontal (1 - 1) and other in vertical direction (2 - 2) are considered (Fig. 3) to analyse the head distribution and flux variation across these sections for each scenario. The FEM simulation was coded in MATLAB and checked for a mass balance for correctness of solutions.

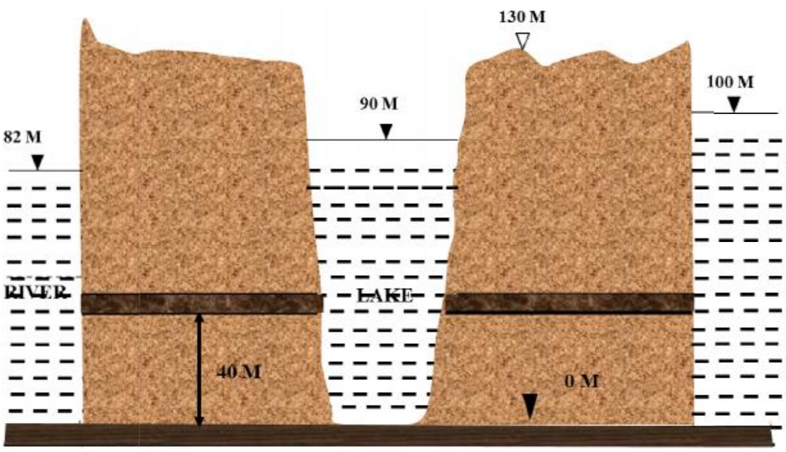


Figure 2. Sectional view of Aquifer Domain across section X-X

Table 1. Zonation of Transmissivity (source: CGWB 2011)

Zone No.	Tx (m^2/d)	Ty (m^2/d)	Multiplying factor for aquitard recharge (e)	Zone No.	Tx (m^2/d)	Ty (m^2/d)	Multiplying factor of aquitard recharge (e)
1	2314	1000	0.78	8	202	150	0.56
2	1638.73	1400	0.74	9	757	576	0.68
3	85.53	75	0.54	10	835.14	445	0.70
4	954	684	0.72	11	700	600	0.66
5	450	220	0.64	12	6725	3000	0.80
6	415	250	0.62	13	250	100	0.58
7	335	300	0.60	14	85.53	75	0.54

III. RESULTS AND DISCUSSIONS

A. Scenario I – Comparison between a constant river head and linearly varying river head boundary

The influence of the river boundary condition on the aquifer head simulation is studied by considering the linear variation of river head (2m between upstream and downstream) along the river length. However first the sensitivity to constant river head condition is examined by changing river head of 82 m to 78, 80, 84, 88, and 90 m respectively. Figure 4 shows the groundwater contours for the constant river head condition whereas contours plot for its replacement as linear river head variation condition are depicted in Figure 5. The contours are drawn at the end of 50 days simulation period. Figure 6 and 7 displays the three dimensional contour plot for figure 5 and figure 6 respectively.

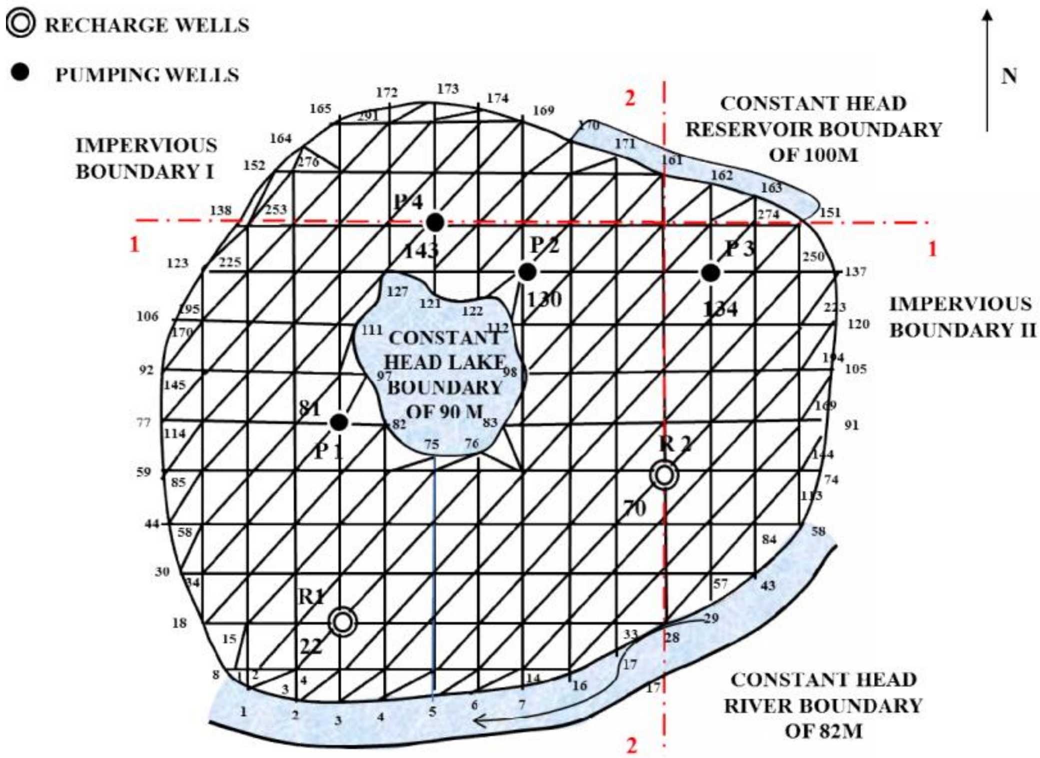


Figure 3. FEM domain discretization

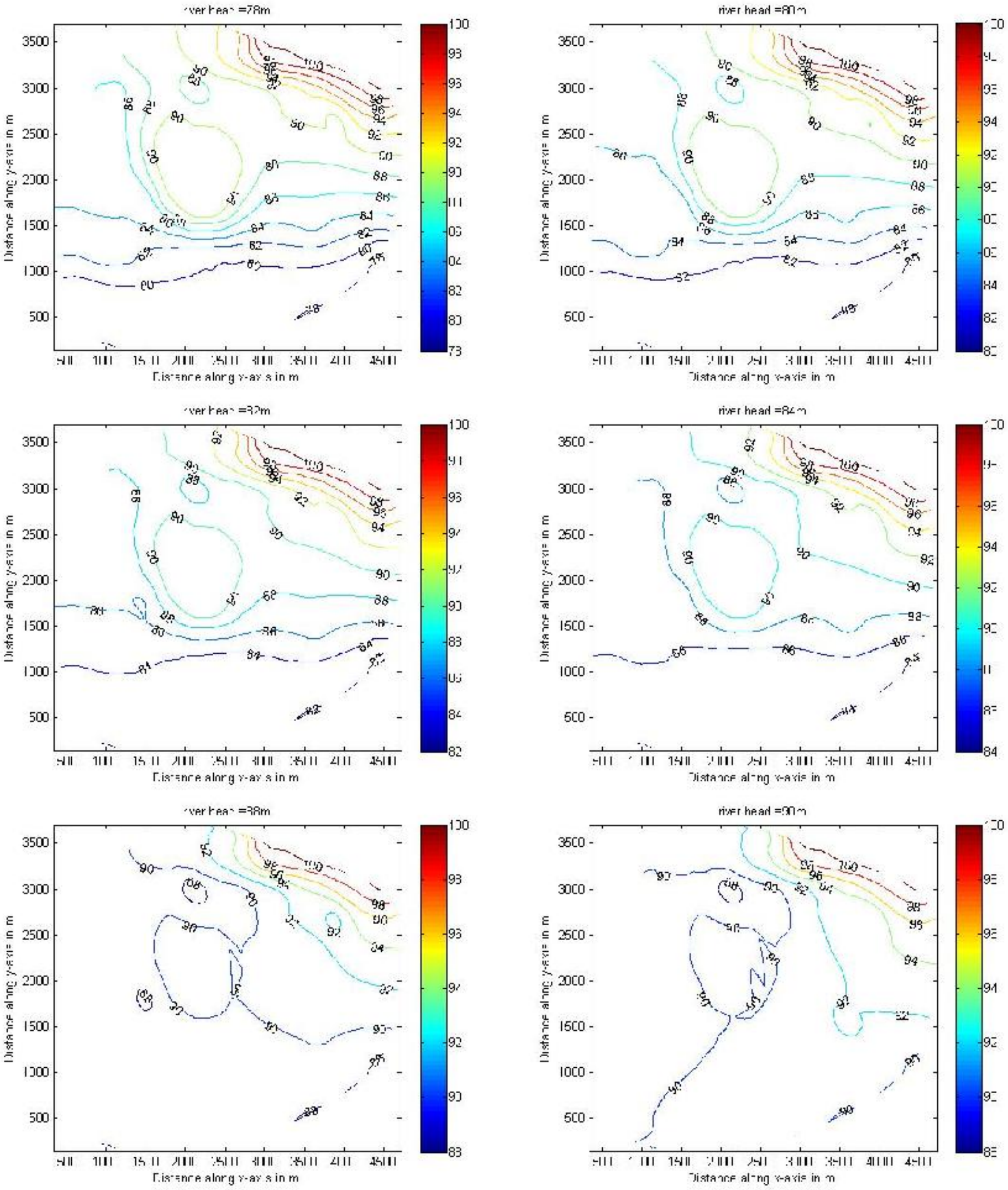
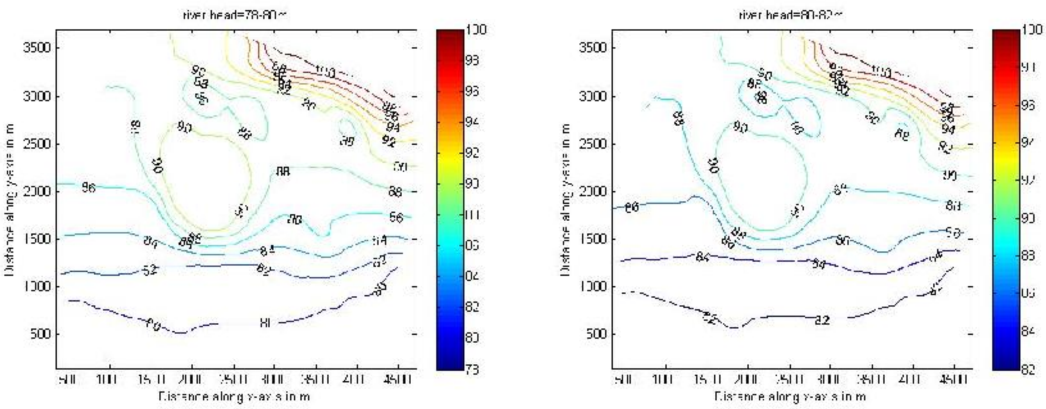


Figure 4. Surface contour plot of transient state groundwater head distribution for different constant river head values after 50 days of simulation



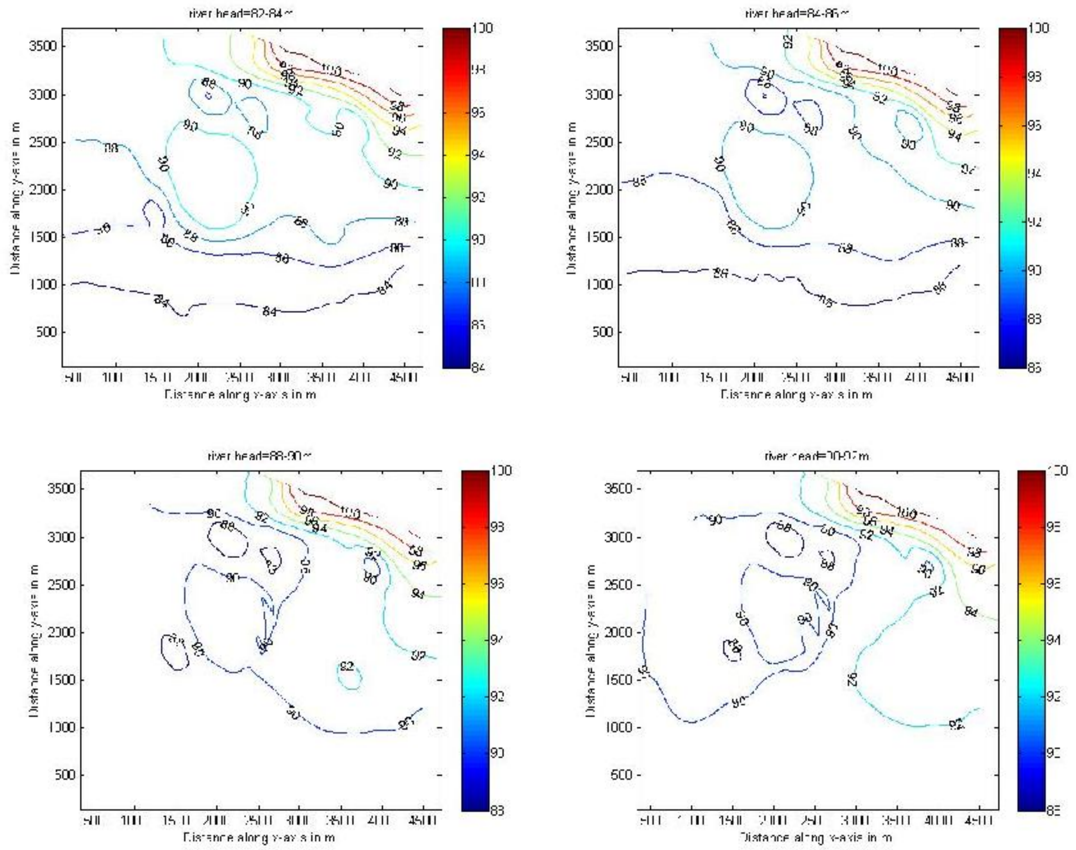
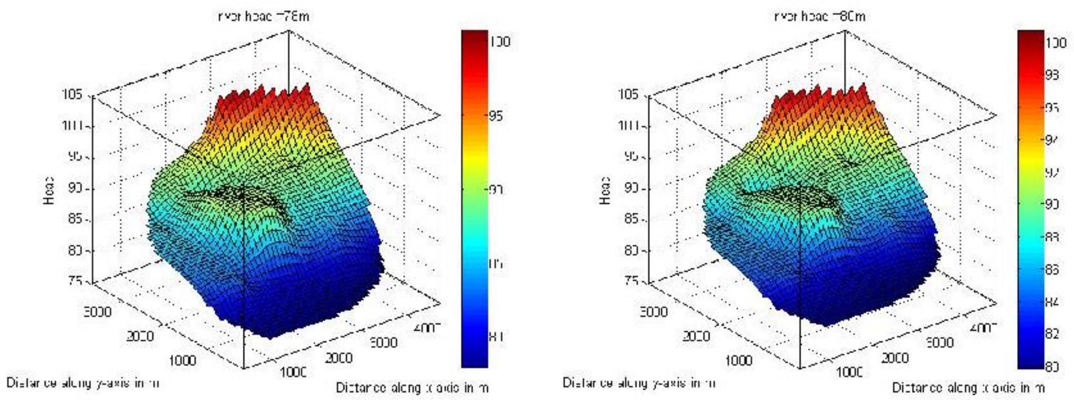


Figure 5. Surface contour plot of transient state groundwater head distribution for different linearly varying river head values after 50 days of simulation

Lower head gradient near the river boundary is observed in figure 5 due to the linear variation of river head compared to figure 4. This is attributed to the decreased groundwater flow rates towards the river for the linear variation of river head. The regions that are away from the river boundary are least influenced by the linear variation of river head. The region of influence of the pumping wells and injection wells is found to be reducing with the increase in the river head values (Fig. 4). However reduction occurs at a slower rate in figure 5 because of the linear variation of river head. It is observed that the effect of linear variation of river head is less effective away from the river boundary which is expected.



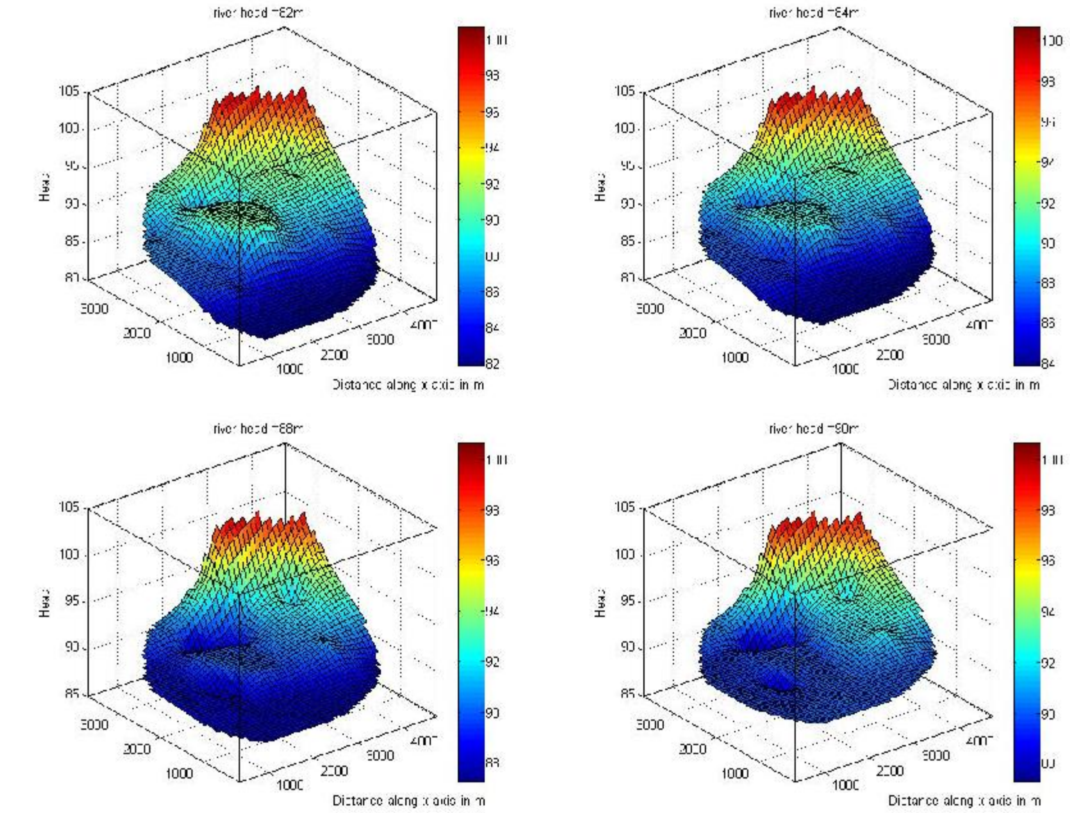
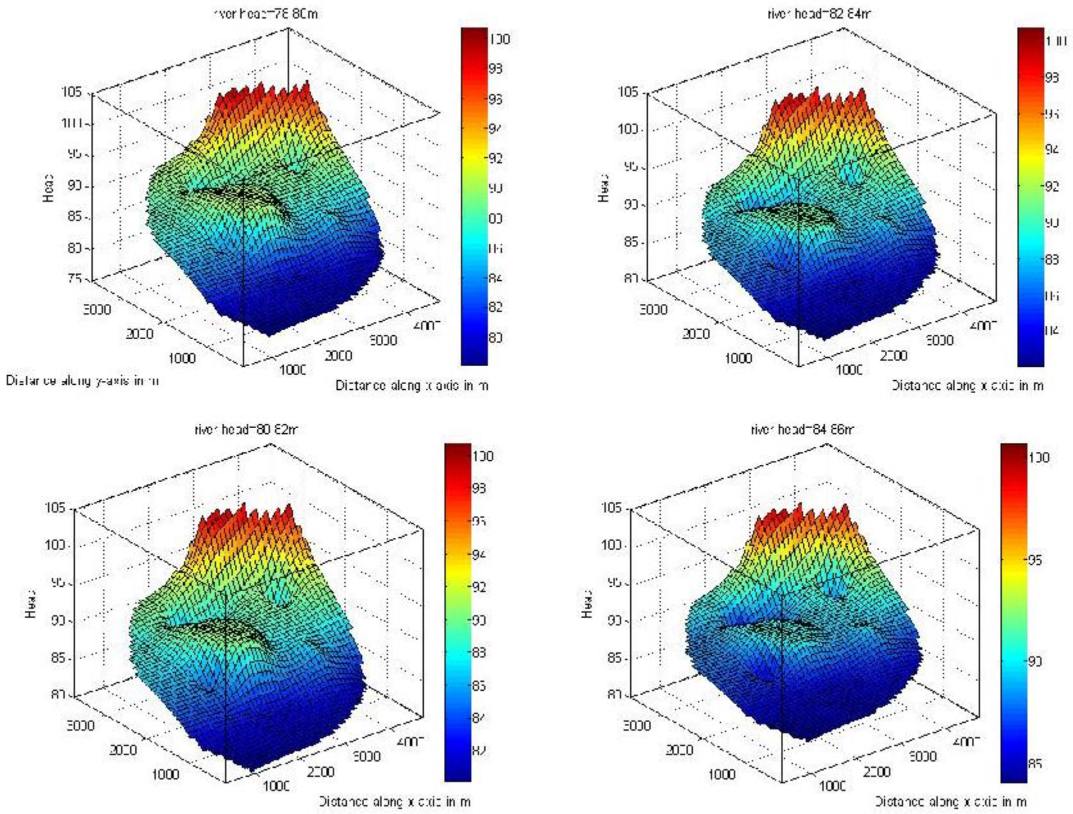


Figure 6. Transient state groundwater 3D contour plot of head distribution for different constant river head values after 50 days of simulation



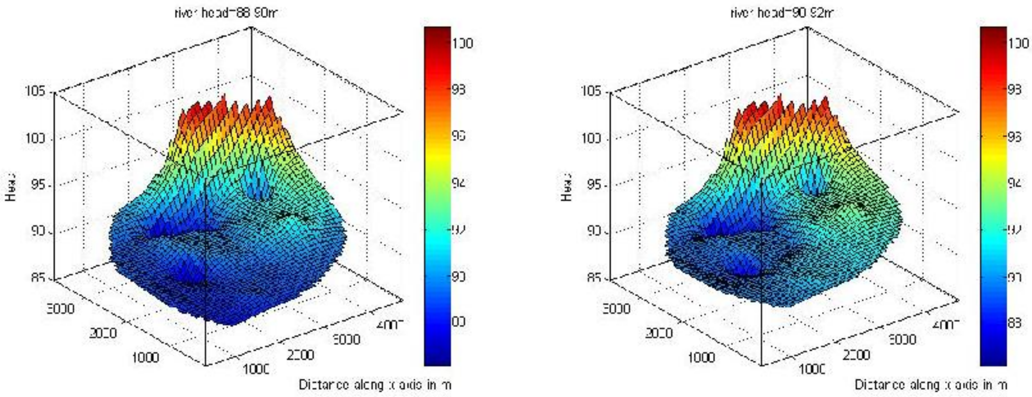


Figure 7. Transient state groundwater 3D contour plot of head distribution for different linearly varying river head values after 50 days of simulation

There is a noticeable increase in the groundwater head especially at node 70 and 22 for the linear variation of the river head from constant value for various ranges since these nodes are closer to the river. With the decrease in the proximity to the river, its effect on the head values also seems diminishing for both the cases.

B. Scenario II – Comparison when the impervious boundary is replaced by an inflow boundary.

A boundary inflow flux rate of 5 m^2/d is considered along the eastern impervious boundary so as to study its influence on the aquifer system. With the application of boundary flux, an increased head is noticed in the north eastern region where the inflow is occurring. It is observed that the introduction of boundary inflow from eastern side has certain effect on the groundwater head values.

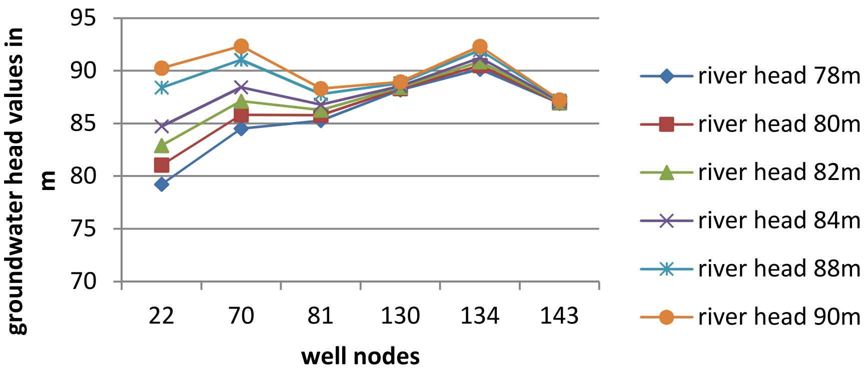


Figure 8. Groundwater heads at the pumping and injection wells for different constant river head values.

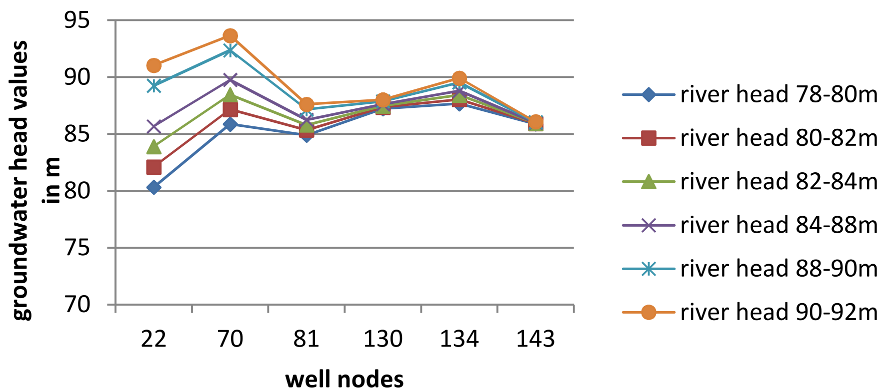


Figure 9. Groundwater heads at the pumping and injection wells for different linearly varying river head values.

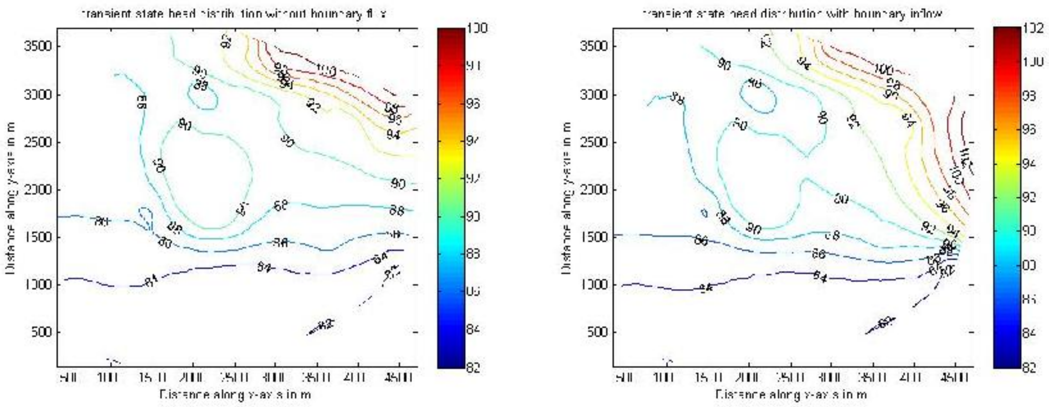


Figure 10. Surface contour plot for transient state head distribution after 50 days of simulation without and with a boundary inflow of 5m^2/d

The contour lines seem to become more regular with the boundary inflow compared to that without boundary flux (Figure.10). The boundary inflow causes increased supply from the surroundings at a faster rate for the compensation of the head decline by pumping and as a result the system attains steady state more rapidly than before.

The transmissivity in the eastern region is 415m^2/d, which is less compared to other zones, might have a role in the higher head. The combined effect of both the constraints may be the cause of the head increase in that zone. The boundary flow influence on head drawdown compensation can be noted from figure 10 at (node 70 and 133) nodes being close to the boundary with flux inflow. The radius of influence around the wells is affected in the contour plot of the aquifer due to boundary inflow. The head drawdown around the pumping wells is reduced with the boundary inflow especially at the wells close to the inflow boundary.

Figure 12 shows the flux variation across section 2-2 without boundary inflow and with a boundary inflow of 5m^2/d. For the latter an increased flux across section 2-2 is observed implying greater flow velocity. With the application of boundary flux, an increase in the groundwater head is observed. At the injection and pumping wells (nodes 70, 130 and 134) the boundary flux inflow causes a change of 0.25%, 0.063%, and 0.418% respectively whereas at nodes 22, 81 and 143 only 0.0024%, 0.0008% and 0.01% change is observed.

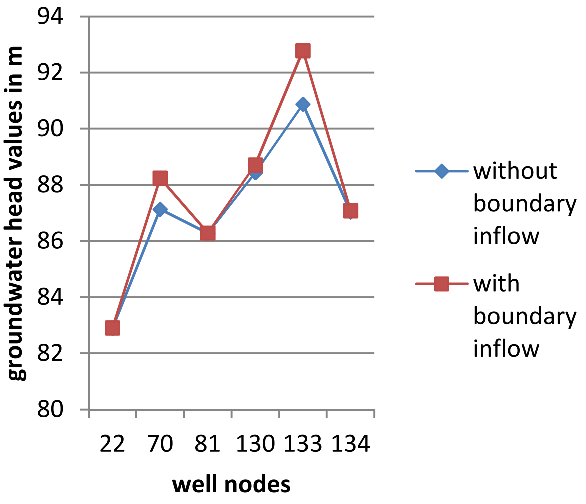


Figure11.Transient state head variation at the pumping and injection wells with and without boundary inflow after 50 days of simulation.

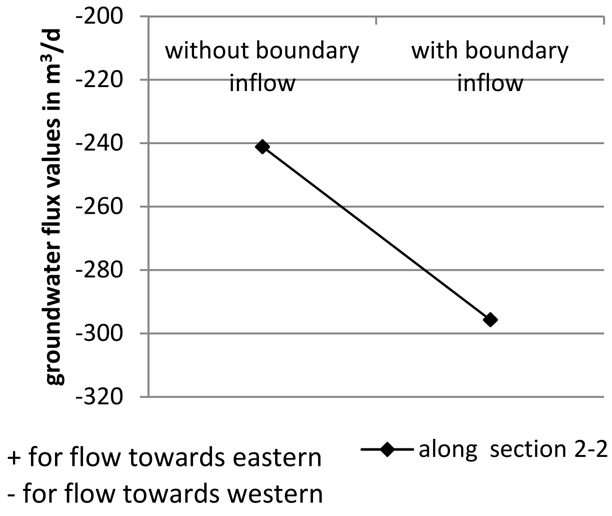


Figure 12. Flux variations along section 2-2 with and without boundary inflow

C. Scenario III – Comparison between homogeneous-isotropic, heterogeneous -isotropic and heterogeneous -anisotropic conditions

Groundwater head distribution in the aquifer under the following conditions is examined for

I. Homogeneous isotropic aquifer (Tx = Ty = 500 m^2/d) under transient state condition after 50 days.
II. Homogeneous anisotropic aquifer (Tx = 250 m^2/d; Ty = 500 m^2/d) under transient state condition after 50 days.

III. Heterogeneous isotropic aquifer (Tx = Ty) under transient state condition after 50 days (Table 1).

IV. Heterogeneous anisotropic aquifer with uniform recharge under transient state conditions after 50 days (Table 1).

V. Heterogeneous anisotropic aquifer with non-uniform recharge (depends on the transmissivity values) by applying a multiplying factor (e) for annual average aquitard recharge of 0.0001519 m/d after 50 days (Table 1).

The above five conditions are simulated using the FEM model and the head values are obtained for each condition. Groundwater head values at the pumping and injection wells for 1st, 2nd, 3rd, 4th and 5th condition are plotted for the 2 injection wells and 4 pumping wells after 50 days in figure 13. The changes in the flux value across section 1-1 under different conditions are compared in figure 14.

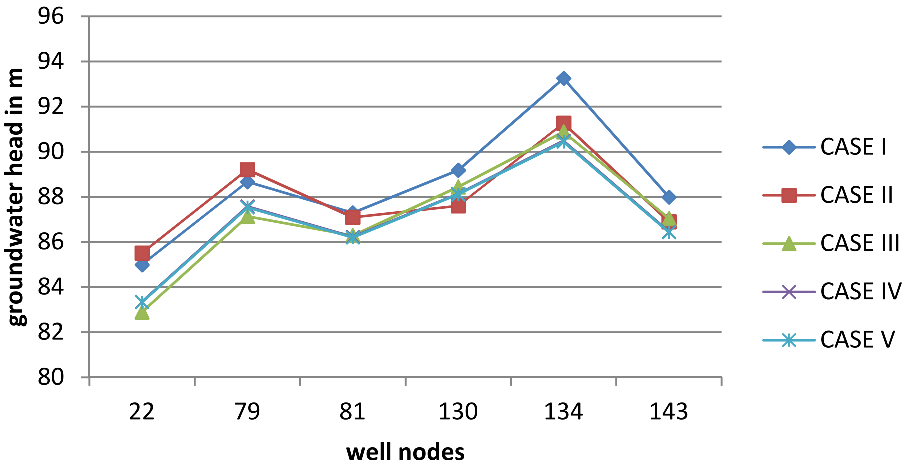


Figure 13. Groundwater heads at pumping and injection wells for case I, II, III, IV & V

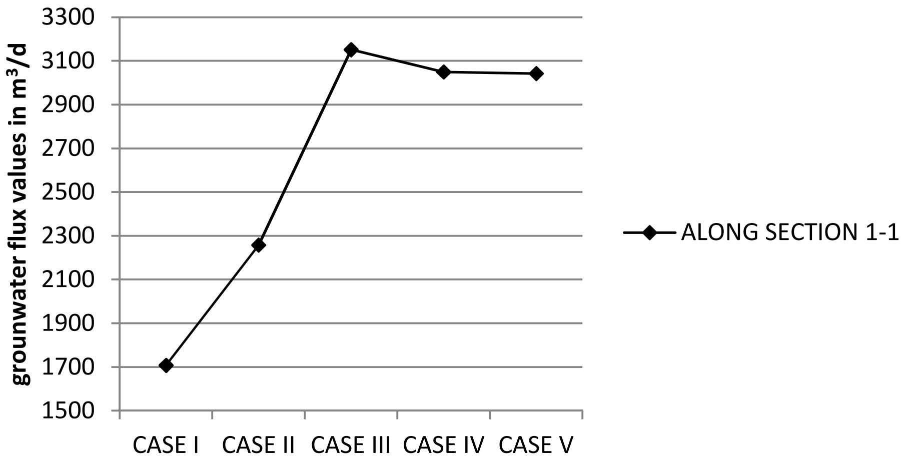


Figure 14. Flux variation along section 1-1 for case I, II, III, IV & V

From the figure 13, it is evident that comparing case I and case II, an increase in the groundwater head values at injection wells and, a decrease in the groundwater head values at pumping wells are observed for the later. Similar scenario is observed between case III, case IV and case V. In general, a rise in groundwater head at the injection wells and a fall in head at the pumping wells for anisotropic condition are observed for both homogeneous and heterogeneous aquifer conditions.

To study the effect of each parameter in detail, the flux across section 1-1 is plotted as shown in figure 14. A more rapid change in the flux value is observed for case III – heterogeneous anisotropic condition among all the five cases. Comparing case I and case II an increased flux is observed when anisotropy is considered for homogeneous aquifer. The plot followed a general trend; more rapid change in the flux value is observed when anisotropic condition is considered.

IV. Conclusions

The applicability of a groundwater model to a real situation depends on the adequacy of the input data and the parameters. For convenience in aquifer simulation often the reality in river head distribution, aquitard recharge, aquifer heterogeneity and influence of the other boundary conditions are simplified. In the present study, a comparison of the practical groundwater flow considerations is carried out by considering a complex heterogeneous confined aquifer flow problem. By transient FEM simulation of the flow domain, the model has been applied with three different scenarios representing the plausible field conditons. It can be conclude from the present study that over simplification of real problem can lead to erroneous groundwater head distribution of the aquifer systems. This may give wrong management decisions since simulation models are called several times in the management models.

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